Rossby Wave and Pure Rotational Wave

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1. Pure inertial wave

Let us consider pure inertial motion of a particle on a frictionless horizontal surface of the rotating earth in a β-plane approximation. If such a particle be projected at the equator with zonal velocity U and meridional velocity infinitesimally small compared to U, then the trajectory of the projectile will be a sinusoidal curve repeatedly cutting the equator, and having wavelength L_T and period T given by

$$L_T = 2\pi\sqrt{\frac{U}{\beta}}, \qquad (1)$$

$$T = \frac{2\pi}{\sqrt{U\beta}}. \qquad (2)$$

It can also be shown, along the path of the projectile, that $(V/R_T)+f$ is conserved, where V is its total velocity, R_T is its radius of curvature and f is the value of Coriolis parameter, i.e., $f=\beta y$, y being the distance of the particle from the equator. In other words,

$$\frac{d}{dt}\left(\frac{V}{R_T}+f\right)=0. \qquad (3)$$

If a set of suitably synchronized particles be projected from the equator in a similar way, then to the observer standing on earth, there will appear a wave moving with a velocity c given by

$$U-c=\frac{BL_sL_T}{4\pi^2}, \qquad (4)$$

where L_s is the wavelength of the wave form, L_T and L_s being connected by

$$\frac{L_T}{L_s}=\frac{U}{U-c}. \tag{5}$$

This wave can be called a pure inertial wave or pure rotational wave because it is only the earth's rotation which has played the part of a restoring force, while other forces such as earth's gravitation or elastic forces have played no part whatsoever.

2. Rossby wave

When we deal with a real fluid on a rotating earth, we have to deal with not only the earth's rotation but also with the earth's gravitation which plays its part through the quasi-static pressure field inside the fluid. Under simplest conditions, if we visualize a flow which has no shear normal to the flow lines, then we have the well-known Rossby (1939) wave formulas:

$$\frac{d}{dt}\left(\frac{V}{R_s}+f\right)=0, \tag{6}$$

$$U-c=\frac{\beta L_s^2}{4\pi^2}. \tag{7}$$

There is a clear distinction between the waves described in Sections 1 and 2. In the wave of Section 1, earth's rotation is the sole agent controlling the motion. In the wave of Section 2, earth's rotation is important but it is not the exclusive and sole agent; earth's gravity also plays its part so much so that for very large wavelengths, it may become nearly impossible to distinguish between Rossby waves and the so-called gravity waves on a rotating earth (Matsuno, 1966). Thus, *pure gravity waves in the absence of rotation make one class of waves*; *pure rotational waves without the role of gravity make another class of waves. Rossby waves form an intermediate class having features of both these classes of waves, being closer to pure rotational waves than to pure gravity waves.*

3. C.A.S. trajectory

Eqs. (3) and (6) both relate to an individual parcel and hence are satisfied along the trajectory of a parcel. In (3), the curvature of the trajectory and hence the spin of the parcel itself is involved; in other words, an individual parcel conserves its own absolute spin. In (6), the curvature of the streamline and hence the vorticity of the instantaneous flow is involved; in other words, absolute vorticity of the flow field has a constant value along the trajectory of a parcel. If it is appropriate to call the latter a C.A.V. (Constant Absolute Vorticity) trajectory, it may not be out of place to call the former a C.A.S. (Constant Absolute Spin) trajectory.

It seems pertinent to ask here why text-book descriptions of a C.A.V. trajectory are often associated with a stationary Rossby wave (Haltiner and Martin, 1957, p. 353; Petterssen, 1956, p. 147). The reason appears to be that after Rossby *et al.* (1939) had derived his wave velocity formula (7) which was based on Eq. (6), he (Rossby, 1940) attempted to reconcile it with the concept of Eq. (3), this reconciliation only being possible for a stationary Rossby wave. Rossby (1940) also simultaneously introduced the phrase C.A.V. trajectory. Ever since that time, C.A.V. trajectories have often been associated with stationary waves in the meteorological literature, and there is some confusion (Panofsky, 1958) between C.A.V. and C.A.S. trajectories.

A C.A.V. trajectory is different from a C.A.S. trajectory; neither of the two trajectories need be associated with stationary waves. However, C.A.V. and C.A.S. trajectories become identical only for a stationary Rossby wave.

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